Firewalls and the Quantum Properties of Black Holes

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by

Dylan Louis Veyrat

Advisor: Marc Sher

Senior Research Coordinator: Gina Hoatson

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Abstract

With the proposal of black hole complementarity as a solution to the information paradox resulting from the existence of black holes, a new problem has become apparent. Complementarity requires a violation of monogamy of entanglement that can be avoided in one of two ways: a violation of Einstein's equivalence principle, or a reworking of Quantum Field Theory [1]. The existence of a barrier of high-energy quanta - or "firewall" - at the event horizon is the first of these two resolutions, and this paper aims to discuss it, for Schwarzschild as well as Kerr and Reissner-Nordström black holes, and to compare it to alternate proposals.

1 Introduction, Hawking Radiation

While black holes continue to present problems for the physical theories of today, quite a few steps have been made in the direction of understanding the physics describing them, and, consequently, in the direction of a consistent theory of quantum gravity. Two of the most central concepts in the effort to understand black holes are the black hole information paradox and the existence of Hawking radiation [2].

Perhaps the most apparent result of black holes (which are a consequence of general relativity) that disagrees with quantum principles is the possibility of information loss. Since the only possible direction in which to pass through the event horizon is in, toward the singularity, it would seem that information entering a black hole could never be retrieved. In other words, while one could predict the evolution of a quantum system involving a black hole in the forward direction of time, a later state of a black hole could not be used to calculate the state of the system at some past time: several different past evolutions could result in identical black holes. This violates the quantum principle of unitarity [3].

In a 1974 paper, Stephen Hawking theorized that black holes are not completely black and do in fact emit radiation, resulting from quantum processes near the event horizon [2]. Black holes as thermodynamic systems have temperature, as well as an entropy proportional to their surface area. The Hawking radiation, which corresponds to thermodynamic blackbody radiation, radiates power P from a chargeless, non-rotating black hole of mass M given by the Stefan-Boltzman-Schwarzschild-Hawking power law [4]:

$$P = \frac{\hbar c^6}{15360 \pi G^2 M^2}, \tag{1}$$

where $\hbar$ is the reduced Planck constant, c is the speed of light in a vacuum, and G is the gravitational constant. This is effectively the rate of mass loss, and taking this as a differential equation for the black hole's mass as a function of time, one can solve for the time it takes for the black hole to evaporate:

$$-c^2 \frac{dM}{dt} = P \tag{2}$$

$$-\int_{t_0}^{t} M^2 dM = \int_{t_0}^{t} \frac{\hbar c^4}{15360\pi G^2} dt \tag{3}$$

$$-\frac{M^3}{3} = \frac{\hbar c^4 t}{15360\pi G^2} - \frac{M_0^3}{3} \tag{4}$$

where M_0 is the initial mass. Then, setting $M = 0$ and solving for t gives

$$t_{evap} = \frac{5120\pi G^2 M_0^3}{\hbar c^4}. \tag{5}$$

While this radiation does allow a black hole to dissipate, it was not initially believed to carry information from the black hole. The information paradox still stood, but Hawking radiation has played a part in attempting to resolve this problem.

2 Entanglement and Entropy

The process of Hawking radiation involves the creation of particle-antiparticle pairs from vacuum fluctuations near the event horizon of a black hole. These pairs usually proceed to annihilate, but occasionally one of the pair escapes the gravity of the black hole, being emitted as Hawking radiation, and the other falls in. Since the total energy of the infalling particle of such a "Hawking pair" is negative (due to the gravitational field), the black hole loses mass in this process [2].

This is where entanglement enters the picture. The particle pair created is analagous to the well-known example of two entangled spin-$\frac{1}{2}$ particles, whose total spin we know is 0, without knowing the individual spins of either particle until a measurement is preformed on one or the other.

Entanglement is related to the entropy of the quantum system in question, and the discussion here will become more relevant after a discussion on complementarity in section 3. For now, we develop the concept of subadditivity of entropy, which is appropriate to the system of a Hawking pair.

First, consider a particle whose state $|\psi\rangle$ in the Hilbert space $\mathcal{H}$ of possible states for this particle can be given as a linear combination of "pure" states, or eigenstates $|\psi_i\rangle$:

$$|\psi\rangle = \sum_i c_i \, |\psi_i\rangle. \tag{6}$$

A particle in a pure state, of course, would have all coefficients but one equal to 0. The state of this particle has an associated density matrix ρ [5] with elements (in an orthonormal basis $|u_n\rangle$ of $\mathcal{H}$) given by

$$\rho_{mn} = \sum_i c_i \, \langle u_m|\psi_i\rangle \, \langle \psi_i|u_n\rangle. \tag{7}$$

The fact that the coefficients c_i are essentially measures of the probability of finding the particle in a particular state $|\psi_i\rangle$ leads one to consider an expression similar to that of the Gibbs entropy in classical statistical mechanics [6]:

$$S(\rho) = -\operatorname{tr}(\rho \ln \rho). \tag{8}$$

This is the von Neumann entropy, which can also be expressed in terms of the coefficients c_i as

$$S = -\sum_i c_i \ln c_i. \tag{9}$$

The von Neumann entropy is clearly 0 for pure states, where all c_i in the sum are either 0 or 1.

Now the a quantum expression for entropy has been developed, subadditivity of entropy can be presented. Given two particles, A and B, with state vectors for the particles and for the system of both in the Hilbert spaces $\mathcal{H}^A$, $\mathcal{H}^B$, and $\mathcal{H}^A \otimes \mathcal{H}^B$, respectively, the following inequality holds for the entropies of the three states:

$$S(\rho^{AB}) \leq S(\rho^A) + S(\rho^B). \tag{10}$$

This is the subadditivity of entropy [7] and it is evidently true in the case of two entangled spin-$\frac{1}{2}$ particles. There is no uncertainty in what value a measurement of the spin of the overall system will return, but neither particle has a definite spin. Indeed, a situation like this, where the system as a whole is in a pure state and $S(\rho^{AB}) = 0$, is said to be "maximally entangled".

Complementarity will now be introduced, after which this section will be much more relevant in discussing strong subadditivity and the Page time.

3 Black Hole Complementarity

A proposed resolution of the information paradox is the concept of black hole complementarity, which can be stated as follows: the observed evolution of information falling into a black hole is dependent on whether the observation is in an external frame of reference (that is not falling into the black hole) or in the frame of reference falling with the information, and these two different observed outcomes are consistent with each other [3].

In the reference frame of falling into the black hole, the information passes through the horizon, experiencing ordinary free fall, and eventually reaches the singularity. On the other hand, in the external frame, gravitational time dilation effects near the horizon make it appear as if the infalling information never reaches the horizon. Instead, it approaches it ever more slowly, being spread out uniformly over time across a surface at an arbitrarily small distance outside the horizon. This surface is known as the stretched horizon.

As the infalling information becomes a part of this stretched horizon, it adds energy to the stretched horizon and heats it up. This then causes the stretched horizon to emit radiation, which corresponds to the Hawking radiation discussed in section 1 (see figure 1). This radiation does in fact carry information away from the black hole.

Each of the two processes described is unitary in its own frame; in other words, information is lost in neither. A consequence of this theory, however, is that any quantum of Hawking radiation emitted from a sufficiently old black

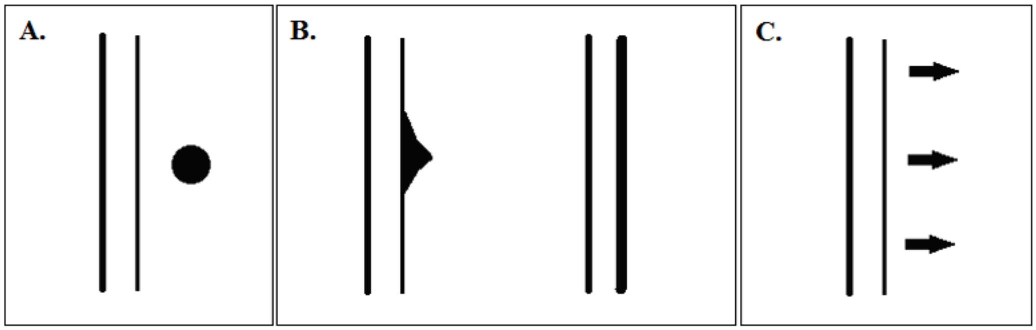

Figure 1: A. Information falling toward a black hole's event horizon. B. That information being absorbed into and extended over the stretched horizon in an external reference frame. C. The information being re-emitted as Hawking radiation.

hole (one theorized to be roughly past half of its lifetime) must be entangled with the system of all Hawking radiation that black hole has emitted in the past [8] (as will be discussed further in section 4). The necessity of this entanglement presented by complementarity proves to be a problem.

4 Entropy Revisited, the Page Time

While the subadditivity of quantum entropy is helpful in understanding entanglement between two systems, there exists another entropy relation that involves the entropies three systems. This time the individual Hilbert state spaces are $\mathcal{H}^A$, $\mathcal{H}^B$, and $\mathcal{H}^C$ and there exist also combined state spaces $\mathcal{H}^A \otimes \mathcal{H}^B$, $\mathcal{H}^A \otimes \mathcal{H}^C$, $\mathcal{H}^B \otimes \mathcal{H}^C$, and $\mathcal{H}^A \otimes \mathcal{H}^B \otimes \mathcal{H}^C$. The state of each system or group of systems in the relevent Hilbert space again has an associated density matrix, and consequently a von Neumann entropy. A basic principle of quantum information theory is that these entropies satisfy [7]

$$S(\rho^{ABC}) + S(\rho^{B}) \leq S(\rho^{AB}) + S(\rho^{BC}). \tag{11}$$

This is the principle of strong subadditivity of quantum entropy.

A relevent consequence of this inequality is in the case of entanglement in a system of three particles. If a particle B is maximally entangled with two independent particle A and C, it follows that $S(\rho^{AB}) = S(\rho^{BC}) = 0$. This cannot be true, however, because $S(\rho^{B}) > 0$; the strong subadditivity of entropy thus brings us to the concept of monogamy of entanglement. A particle cannot be simultaneously entangled with two independent systems. This, along with another quantum property of black hole evaporation, leads to the firewall theory.

Before the proposal of black hole complementarity, it was theorized that the quantum entropy of all Hawking radiation emitted by a black hole keeps increasing as the black hole evaporates, eventually reaching the original entropy of the black hole. This would be a violation of unitarity, however, and a new model accompanied the theory of complementarity. Instead, the entropy of the Hawking radiation steadily increases until the so-called Page time, roughly 54% of the way through the black hole's lifetime [9]. At that point, all subsequent radiation (called late radiation) is theorized to be emitted entangled with all previously emitted radiation (or early radiation). The result is a decline in the entropy of Hawking radiation, and no violation of unitarity. A new problem arises from the entanglement, however, and the theory of the firewall aims to resolve it.

5 The Firewall Resolution

The mechanism that leads to Hawking radiation is the creation of particle-antiparticle pairs in the vacuum near the event horizon of a black hole; one member of the pair, with negative energy, falls in, and the other escapes, causing a loss of mass for the black hole. The two members of each pair of particles produced this way are entangled with each other. This, combined with the entanglement established in section 4, violates the principle of monogamy of entanglement. A particle cannot be simultaneously entangled with two independent systems (here, the other particle in the pair and all past Hawking radiation).

One way to resolve this violation is by allowing for "breaking" of the entanglement of the particle-antiparticle pair [10]. This releases a significant amount of energy, and the result is a barrier of high-energy quanta just below the event horizon of the black hole. This is the theory of the firewall, which is one of several possible resolutions of the problem raised by complementarity.

Another way of explaining the firewall solution is to consider the entangled particle-antiparticle pair produced near the horizon. For there to be no more entanglement between a particle within the horizon and another outside of it, there must be a significant difference between the quantum fields inside and outside. This would imply a quite significant gradient in the quantum fields at the event horizon of the black hole.

The existence of firewalls, while it would prevent the forbidden double entanglement of late Hawking radiation, would also violate Einstein's equivalence principle, a fundamental base for the well-tested general relativity. By this principle, an observer falling into a black hole should experience "no drama" when passing through the event horizon; free fall through the horizon should be (neglecting tidal effects) indistinguishable from being in an inertial frame of reference in flat spacetime. A firewall at the horizon would certainly distinguish falling into a black hole from such an inertial frame. Other resolutions ensuring that the equivalence principle is not violeted, have problems of their own, however.

6 Rotating and Charged Black Holes

All discussion up until this point has been regarding Schwarzschild black holes, with angular momentum $J = 0$ and charge $Q = 0$, but charged and rotating black holes may also be of interest. The spacetime of the vacuum surrounding a non-rotating, charged (or Reissner-Nordström) black hole is characterized by the Reissner-Nordström metric, given by

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right) c^2 & 0 & 0 & 0 \\ 0 & -\left(1 - \frac{r_s}{r} + \frac{r_Q^2}{r^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix} \tag{12}$$

in polar coordinates, where $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius and

$$r_Q^2 = \frac{Q^2 G}{4\pi\epsilon_0 c^4} \tag{13}$$

(using, as I will continue to do, the $(+,-,-,-)$ signature for temporal and spatial coordinates). The only result of adding charge to a black hole, it seems, is to change the location of its event horizon, and accordingly the firewall theory still applies. The charged black hole does however radiate energy at a different rate on account of this new event horizon; the new power law is [11]

$$P = \frac{\hbar c^6 \left(1 - \frac{Q^2}{4\pi\epsilon_0 GM^2}\right)^2}{240\pi G^2 M^2 \left(2 + 2\sqrt{1 - \frac{Q^2}{4\pi\epsilon_0 GM^2}} - \frac{Q^2}{4\pi\epsilon_0 GM^2}\right)^3}, \tag{14}$$

which, since the Page time is dependent on the lifetime of the black hole, would result in a modified Page time.

Rotating, or Kerr, black holes have a more interesting effect on spacetime. The Kerr metric [12] for a black hole with angular momentum J can be written

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{r_s r}{r^2+a^2\cos^2\theta}\right) c^2 & 0 & 0 & \left(\frac{r_s r a \sin^2\theta}{r^2+a^2\cos^2\theta}\right) c \\ 0 & -\frac{r^2+a^2\cos^2\theta}{r^2-r_s r+a^2} & 0 & 0 \\ 0 & 0 & -(r^2+a^2\cos^2\theta) & 0 \\ \left(\frac{r_s r a \sin^2\theta}{r^2+a^2\cos^2\theta}\right) c & 0 & 0 & -\left(r^2+a^2+\frac{r_s r a^2 \sin^2\theta}{r^2+a^2\cos^2\theta}\right)\sin^2\theta \end{pmatrix}, \tag{15}$$

where $a = \frac{J}{Mc}$. Evidently, the off-diagonal elements $g_{t\phi}$ are non-zero, and the result is an interesting phenomenon called frame dragging: given a particle moving radially inward in the plane of rotation with velocity $\frac{dr}{d\tau} = -v$, one can solve the geodesic equation $\frac{d^2x^\mu}{d\tau^2} = -\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$ (where the Christoffel symbols are $\Gamma^\mu_{\alpha\beta} = \frac{1}{2}g^{\mu\kappa}(\partial_\alpha g_{\beta\kappa} + \partial_\beta g_{\alpha\kappa} - \partial_\kappa g_{\alpha\beta})$ as usual) to calculate the angular push $\frac{d^2\phi}{d\tau^2}$ that the particle "feels" in the direction of rotation:

In this case, we are only concerned with the symbols $\Gamma^\phi_{\alpha\beta}$. Additionally, since the motion under consideration is radially inward, $\frac{d\theta}{d\tau} = \frac{d\phi}{d\tau} = 0$, and the only remaining Christoffel symbols to compute are Γ^ϕ_{rr}, Γ^ϕ_{tr}, and Γ^ϕ_{tt} (note that $\Gamma^\mu_{\alpha\beta} = \Gamma^\mu_{\beta\alpha}$ as a result of the symmetric spacetime metric). Further, since the only nonzero $g^{\phi\kappa}$ are $g^{\phi\phi}$ and $g^{\phi t}$, and since the only nonzero partial derivatives of metric components are with respect to r and θ, we have only to consider the following two Christoffel symbols:

$$\Gamma^\phi_{rr} = \frac{1}{2}g^{\phi t}(\partial_r g_{rt} + \partial_r g_{rt}) + \frac{1}{2}g^{\phi\phi}(\partial_r g_{r\phi} + \partial_r g_{r\phi}), \tag{16}$$

which, since $g_{rt} = g_{r\phi} = 0$, is zero, leaving us with only one relevant symbol:

$$\Gamma^{\phi}_{tr} = \frac{1}{2} g^{\phi t}(\partial_r g_{tt}) + \frac{1}{2} g^{\phi\phi}(\partial_r g_{t\phi}). \tag{17}$$

The inverse metric can be calculated to give $g^{\phi t} = \frac{r_s r a}{c(r^2+a^2\cos^2\theta)(r^2-r_s r+a^2)}$ and $g^{\phi\phi} = \frac{1}{c^2(r^2-r_s r+a^2)}$, which, combined with the partial derivatives and the fact that $\theta = \frac{\pi}{2}$ since the motion is in the plane of rotation, gives

$$\Gamma^{\phi}_{tr} = \frac{r_s a c}{2r^2(r^2 - r_s r + a^2)}. \tag{18}$$

$\frac{dr}{d\tau} = -v$ is already known, leaving the time dilation factor $\frac{dt}{d\tau}$ to compute from the metric:

$$c^2 = \left(1 - \frac{r_s}{r}\right) c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(\frac{r^2}{r^2 - r_s r + a^2}\right)\left(\frac{dr}{d\tau}\right)^2 \tag{19}$$

$$\frac{dt}{d\tau} = \sqrt{\left(1 - \frac{r_s}{r}\right)^{-1}\left(1 + \frac{r^2 v^2}{c^2(r^2 - r_s r + a^2)}\right)}, \tag{20}$$

from which we can finally calculate

$$\frac{d^2\phi}{d\tau^2} = \sqrt{\frac{\left(1 + \frac{v^2}{c^2}\right) r^2 - r_s r + a^2}{\left(1 - \frac{r_s}{r}\right)(r^2 - r_s r + a^2)^3}} \left(\frac{r_s a c}{r^2}\right) v. \tag{21}$$

Frame dragging, however, does not directly affect the emission of Hawking radiation. There is a different power law associated with Kerr black holes, though, given by [11]

$$P = \frac{\hbar c^6 \left(1 - \left(\frac{Jc}{M^2G}\right)^2\right)^2}{1920\pi G^2 M^2 \left(1 + \sqrt{1 - \left(\frac{Jc}{M^2G}\right)^2}\right)^3}. \tag{22}$$

The Page time is thus affected also in this case.

Another relevant trait of Kerr black holes is the location of the horizon. Besides the obvious singularity at $r = 0$, the metric is singular also where $1/g_{rr} = 0$ and where $g_{tt} = 0$. The first of these condition is satisfied by the surface

$$r = \frac{r_s + \sqrt{r_s^2 - 4a^2}}{2}, \tag{23}$$

which is a spherical event horizon somewhat closer to the singularity than the Schwarzschild radius. The second condition is satisfied by another surface,

$$r = \frac{r_s + \sqrt{r_s^2 - 4a^2 \cos^2\theta}}{2}, \tag{24}$$

an oblate spheroid that touches the event horizon at the rotation poles. The space between the two surfaces is the ergosphere, unique to rotating black holes.

One of the main curiosities of the ergosphere is that within it the t and ϕ coordinates switch, essentially resulting in frame dragging so extreme nothing can rotate opposite the black hole. The ergosphere is not, however, a point of no return; that purpose is still served by a spherical event horizon, which is the inner of the two surfaces calculated above. The main question relevent to the firewall theory here is whether Hawking radiation is emitted from the event horizon, the outer surface of the ergosphere, or somewhere else. Theory suggests [13] that Hawking radiation from Kerr black holes can be understood as a flux that compensates for gravitational anomalies at the horizon, and that Hawking radiation, in the case of a Kerr black hole, still originates at the event horizon. The firewall, if it exists, must then be there, and the ergosphere can still be safely entered and exited.

7 Black Hole Evaporation Times

Equipped with the relevant power laws, it is now possible to calculate the lifetimes of Reissner-Nordström and Kerr black holes. While these laws make for quite complicated differential equations, the process can be made easier by exploiting this fact: given two differentiable functions $f(t)$ and $g(t)$ for which it is true that $\frac{df}{dt}/\frac{dg}{dt} = \frac{f(t)}{g(t)}$, it follows that the ratio $\frac{f(t)}{g(t)}$ is constant. This is not of use quite yet, but it can be easily proven:

$$\frac{df}{dt}/\frac{dg}{dt} = \frac{f(t)}{g(t)} \Rightarrow \frac{df}{dg} = \frac{f}{g} \Rightarrow \int \frac{df}{f} = \int \frac{dg}{g} \Rightarrow \ln f = \ln g + C \Rightarrow \frac{f(t)}{g(t)} = e^C, \forall t. \tag{25}$$

Let us first consider a charged black hole. Defining $\Theta = 1 - \frac{Q^2}{4\pi\epsilon_0 GM^2}$ lets the power law be rewritten

$$P = -\frac{dE}{dt} = \frac{\hbar c^6 \Theta^2}{240\pi G^2 M^2 \left(1+\sqrt{\Theta}\right)^6}. \tag{26}$$

This black hole has rest energy $E_0 = Mc^2$ and energy from its charge given by [14]

$$E_Q = \left(1 - \frac{1}{2}\left(1+\sqrt{\Theta}\right)\right) Mc^2. \tag{27}$$

We now assume that the proportion of Hawking radiation carrying away rest energy is equal to the proportion of the black hole's total energy that is rest energy, and likewise for electromagnetic energy. Specifically:

$$\frac{dE_0}{dt} = -\left(\frac{E_0}{E_0+E_Q}\right) P \qquad , \qquad \frac{dE_Q}{dt} = -\left(\frac{E_Q}{E_0+E_Q}\right) P. \tag{28}$$

It is immediately apparent that $\frac{dE_Q}{dt} / \frac{dE_0}{dt} = \frac{E_Q}{E_0}$, and it thus follows from the beginning of this section that the ratio $\frac{E_Q}{E_0} = 1 - \frac{1}{2}\left(1+\sqrt{\Theta}\right)$ is a constant, and therefore Θ is constant, making the problem of calculating the lifetime much simpler (physically, this means that the ratio $\frac{Q}{M}$ is constant). The equation above for $\frac{dE_0}{dt} = c^2 \frac{dM}{dt}$ can now readily be solved in the same way as the Stefan-Boltzman-Schwarzschild-Hawking power law, but with a multiplicative constant

$$\frac{P_{uncharged}}{P_{charged}} \left(\frac{E_0+E_Q}{E_0}\right) = \frac{\left(\frac{1}{2}+\frac{1}{2}\sqrt{\Theta}\right)^6 \left(\frac{3}{2}-\frac{1}{2}\sqrt{\Theta}\right)}{\Theta^2}, \tag{29}$$

giving a lifetime

$$t_{evap} = \frac{\left(\frac{1}{2}+\frac{1}{2}\sqrt{\Theta}\right)^6\left(\frac{3}{2}-\frac{1}{2}\sqrt{\Theta}\right)}{\Theta^2}t_0 \qquad , \qquad \Theta = 1-\frac{Q^2}{4\pi\epsilon_0 GM^2}, \tag{30}$$

where t_0 is the lifetime of an uncharged, nonrotating black hole with the same mass, as calculated in section 1. The dependence of evaporation time on charge for a black hole of a given mass is plotted in figure 2. Evidently, t_{evap} asymptotically approaches infinity as the black hole approaches an extremal black hole (for which $\frac{Q^2}{4\pi\epsilon_0 GM^2}=1$). Such a black hole is forbidden by Penrose's weak cosmic censorship hypothesis [15], as computing the metric for a black hole with such a charge indicates no event horizon surrounding the singularity, resulting in a naked singularity [11].

In much the same way, for a rotating black hole with angular momentum J, by defining $\Lambda = 1-\left(\frac{Jc}{GM^2}\right)^2$ the power law may be rewritten

$$P = -\frac{dE}{dt} = \frac{\hbar c^6\Lambda^2}{1920\pi G^2M^2\left(1+\sqrt{\Lambda}\right)^3}. \tag{31}$$

By again exploiting the constant ratio between E_0 and the (this time) rotational energy E_J [14]

$$E_J = \left(1-\frac{1}{2}\sqrt{2+2\sqrt{\Lambda}}\right)Mc^2, \tag{32}$$

the lifetime is again found to be a constant multiple of the uncharged, non-

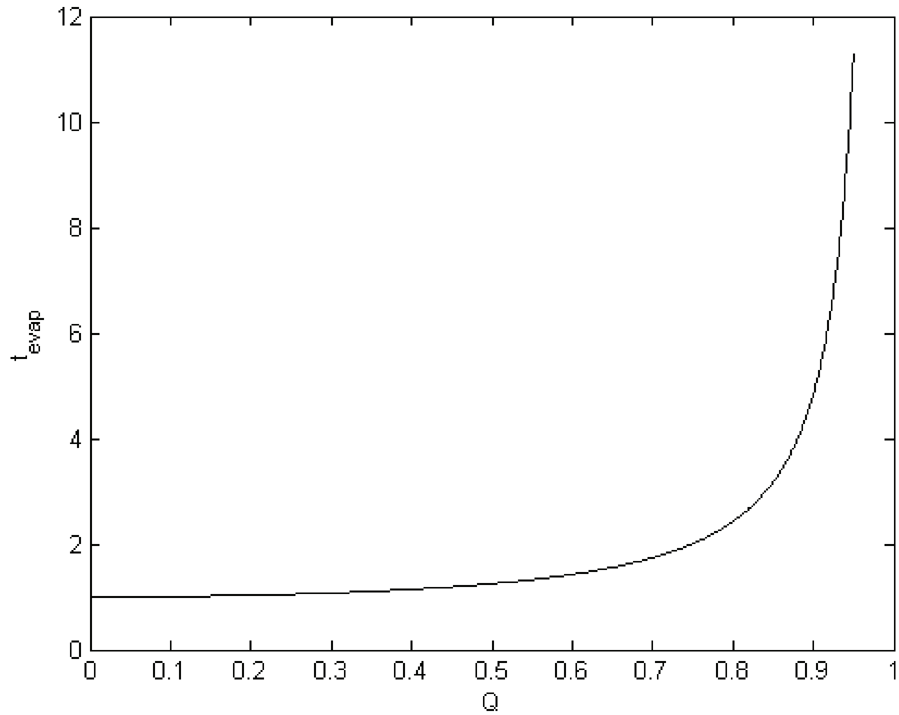

Figure 2: Charge of a Reissner-Nordström black hole (in units of $2M\sqrt{\pi\epsilon_0 G}$) versus its evaporation time as a multiple of t_0.

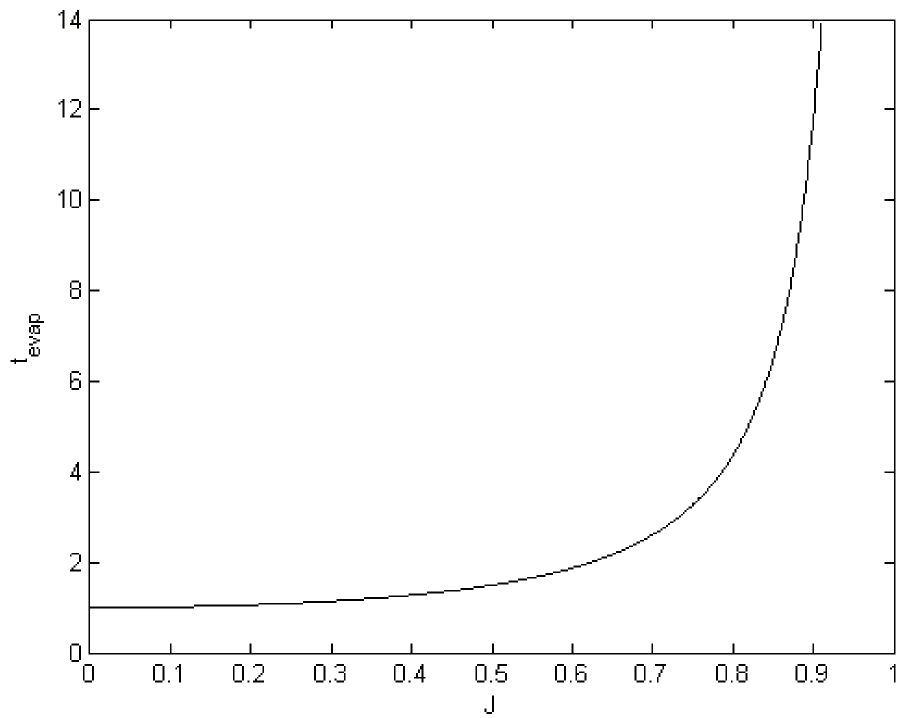

Figure 3: Angular momentum of a Kerr black hole (in units of $\frac{GM^2}{c}$) versus its evaporation time as a multiple of t_0.

rotating evaporation time:

$$t_{evap} = \frac{\left(\frac{1}{2}+\frac{1}{2}\sqrt{\Lambda}\right)^3\left(2-\frac{1}{2}\sqrt{2+2\sqrt{\Lambda}}\right)}{\Lambda^2}t_0 \qquad , \qquad \Lambda = 1-\left(\frac{Jc}{GM^2}\right)^2 . \tag{33}$$

Figure 3 shows the dependence of evaporation time on angular momentum for a black hole of a given mass. Again, t_{evap} asymptotically approaches infinity as the black hole approaches an extremal black hole (in this case, for which $\frac{Jc}{GM^2} = 1$).

8 Other Proposals

Other solutions to the entanglement problem have been proposed. Besides the firewall, there are two other approaches, resulting from the fact that the forbidden entanglement presented by black hole complementarity can be seen as requiring a violation of unitarity, the equivalence principle, or of quantum field theory in its current form [10]. The firewall is the approach where the equivalence principle is violated.

One solution is a lack of entanglement between a quantum of Hawking radiation and all past Hawking radiation. This causes a loss of information and thereby violates unitarity, and in fact is essentially a rejection of the black hole complementarity that was formulated as a solution to information loss in the first place. Another solution involves a reworking of current quantum field theory, in such a way as to allow entanglement (between the particle-antiparticle pair) to be lost more gradually, preventing the existence of a firewall.

There do exist other proposed solutions outside of simply choosing one of three princples to violate. Two of the more prominent proposals are the "fuzzball" picture of black holes in the context of string theory, and the idea that the particle-antiparticle pairs produced near the horizon are connected by wormholes [16].

9 Conclusions

The firewall theory and other resolutions to the entanglement problem resulting from black hole complementarity would be a challenge to verify experimentally, not only because of the lack of readily available, easily observable black holes, but also because they deal with reality within the event horizon, which is theoretically unobservable from the outside universe. Nonetheless, they may have further implications in the thermodynamics of black holes, the dynamics of their collisions, and in future quantum theories of gravity.

The mathematical foundation for the entanglement of quanta of late Hawking radiation with all past Hawking radiation is a rather simplified “toy model” of a black hole system, with a small number of accessible states and a definite formation time after which the black hole absorbs no new matter or energy, simply dissolving through Hawking radiation [8]. This does not account for cases where, for example, two black holes more than halfway through their lifetimes merge into a single black hole with a significantly longer lifetime, which could provide new insight on the issue.

References

[1] Devin, Michael. "Musings on Firewalls and the Information Paradox." *Galaxies* 2 (2014): 189-198.

[2] Hawking, Stephen. "Black Hole Explosions." *Nature* 248 (1974): 30-31.

[3] Susskind, Leonard, Larus Thorlacius, and John Uglum. "The Stretched Horizon and Black Hole Complementarity." *Physical Review D* 48 (1993): 3743-3761.

[4] Manjunath, R. "Force Exerted by Hawking Radiation Emitted from Black Hole." (2010).

[5] Sakurai, J. and Jim Napolitano. "Modern Quantum Mechanics." *Boston: Addison-Wesley* 2 (2011).

[6] Bengtsson, Ingemar and Karol Zyczkowski. "Geometry of Quantum States: An Introduction to Quantum Entanglement." *Cambridge University Press* (2006): 301.

[7] Araki, Huzihiro and Elliot H. Lieb. "Entropy Inequalities." *Communications in Mathematical Physics* 18 (1970): 160-170.

[8] Stoltenberg, Henry and Andreas Albrecht. "No Firewalls or Information Problem for Black Holes Entangled with Large Systems." (2014).

[9] Harlow, Daniel. "Jerusalem Lectures on Black Holes and Quantum Information." (2014).

[10] Almheiri, Ahmed, Donald Marolf, and Joseph Polchinski. “Black Holes: Complementarity or Firewalls?” *Journal of High Energy Physics* 1302 (2013): 62.

[11] Waite, David. “Relativity : A Text Book Tutorial on the Modern Physics of Albert Einstein’s Theories of Special Relativity and General Relativity.” (2005).

[12] Kerr, Roy P. “Gravitational Field of a Spinning Mass as an Example of Algebraically Special Matrices.” *Physical Review Letters* 11 (1963): 237-238.

[13] Murata, Keiju and Jiro Soda. “Hawking Radiation from Rotating Black Holes and Gravitational Anamolies.” (2008).

[14] Lee, Hyun Kyu. “Magnetic Extraction of Rotational Energy of Black Hole.” (2009).

[15] Penrose, Roger. “The Question of Cosmic Censorship.” *Journal of Astrophysics and Astronomy* 20 (1999): 233-248.

[16] Dündar, Furkan. “The Firewall Paradox.” (2014).

[17] Braunstein, Samuel and Stefano Pirandola. “Post-Firewall Paradoxes.” (2014).

[18] Hollowood, Timothy. “Schrodinger’s Cat and the Firewall.” (2014).

[19] Perez, Alejandro. “No Firewalls in Quantum Gravity: the Role of Discreteness of Quantum Geometry in Resolving the Information Loss Paradox.” (2014).