



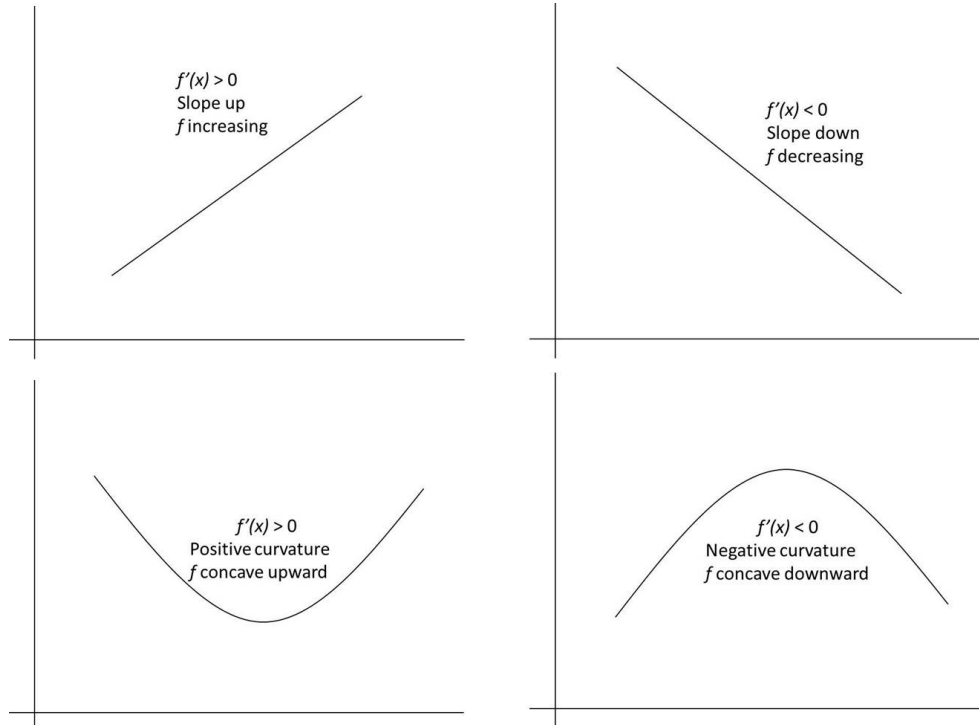
## *Derivatives and Graphing*

(2.2, 2.6, 4.3 and 4.5)

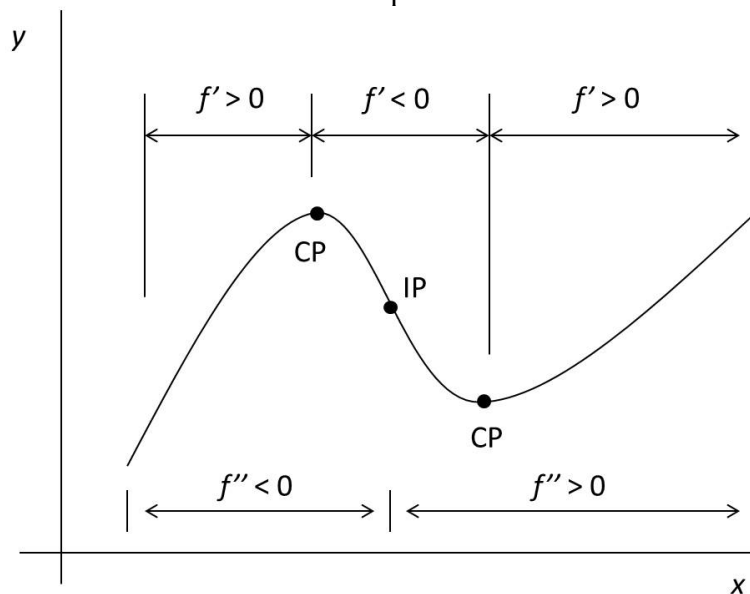
Prelab: Review the definitions of vertical asymptotes (p. 90) and horizontal asymptotes (p.128).

Review Example 7 (p.298) and Example 1 (p. 317).

I. The first derivative tells us about slope and the second derivative tells us about concavity.

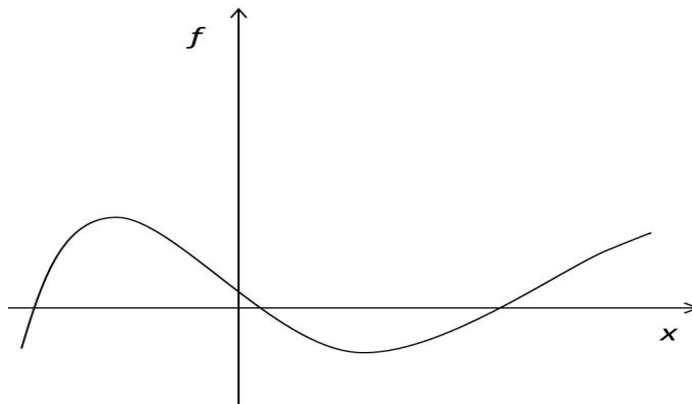


II. Many functions are a combination of different slopes and curvatures.



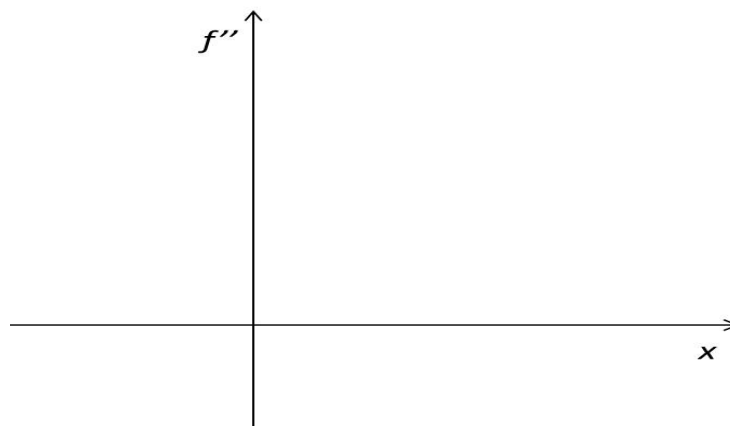
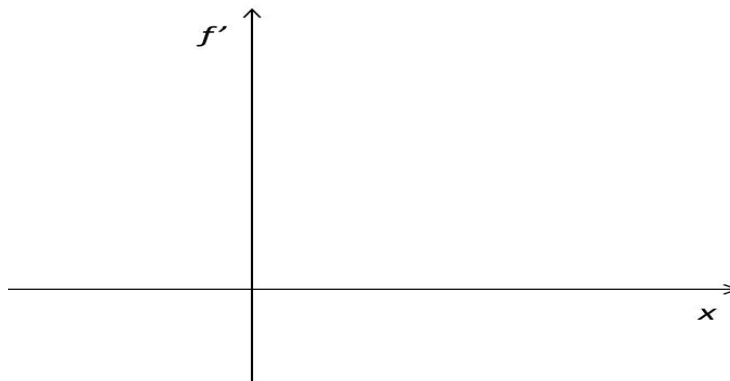
Recall that points on the graph where either  $f'(x) = 0$  or  $f'(x)$  does not exist, are called “critical points” (CP). An inflection point (IP) on a graph is where  $f''(x)$  changes sign.

Example 1: On the graph below, mark slope and curvature using the proper derivative inequality (as seen on the graph above). Label the critical and inflection points.

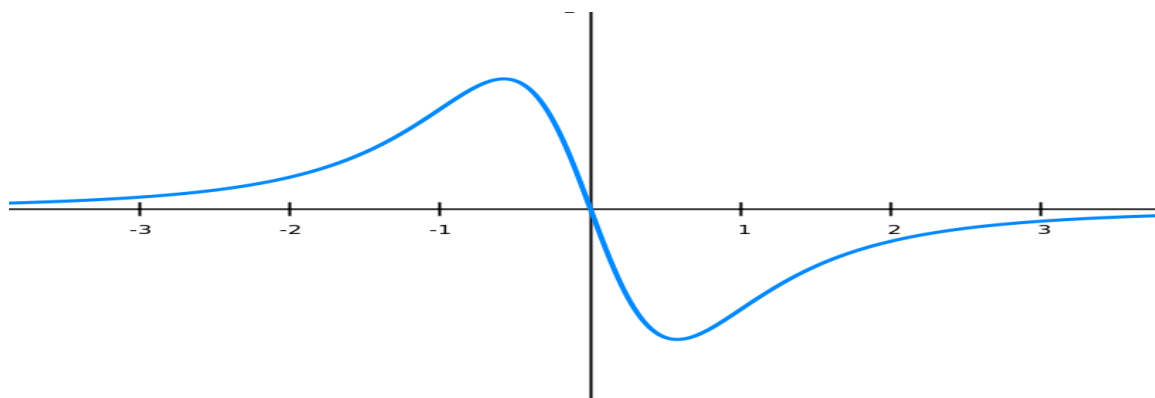


III. Derivative Functions: The previous example gave us a general qualitative description of a graph. As you know, the derivatives of a function are themselves functions, and have their own graphs.

Example 2: Use the information in Example 1 and the graph of the function to sketch graphs of the first and second derivatives.



Example 3: Here is the graph of the derivative  $f'(x)$  of a continuous function  $f(x)$ .



- (a) On what intervals is  $f$  increasing? Decreasing?
- (b) Where does  $f$  have a local maximum? Minimum?
- (c) On what intervals is  $f$  concave upward? Concave downward?
- (d) State where  $f$  has (an) inflection point(s).

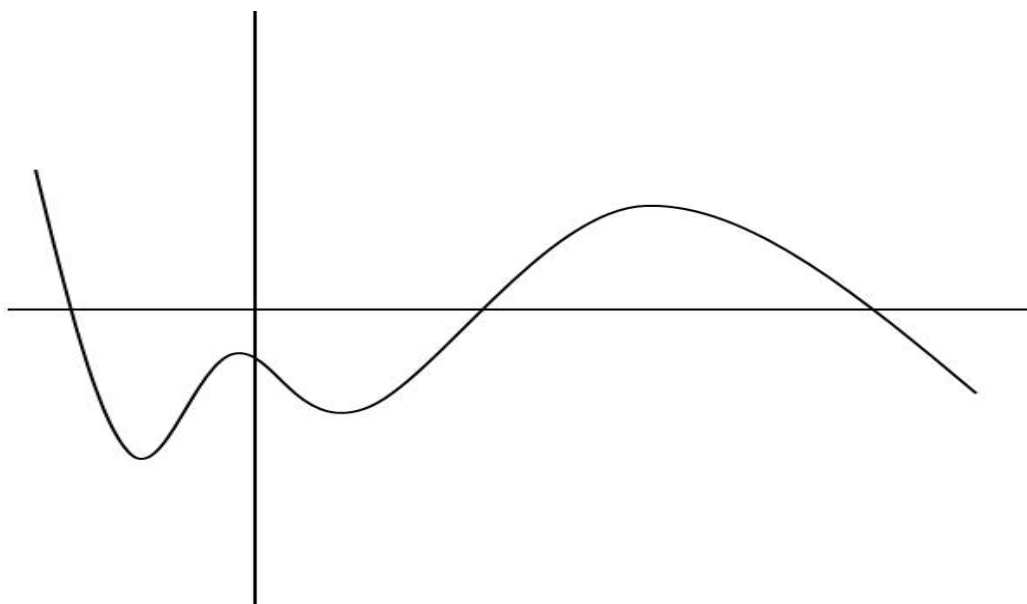
IV. We use the first and second derivatives to locate important graphical features of a function.

Example 4: Consider the function  $f(x) = x - 3x^{2/3}$ . Find (a) intervals of increase/decrease (b) location(s) of maximum(s)/minimum(s) (c) intervals of concavity and (d) location(s) of any IP(s).

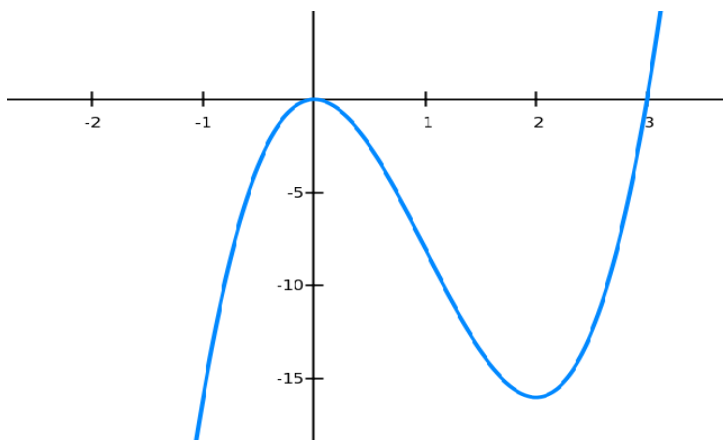
\* We use this information along with the location of roots and the existence of horizontal and vertical asymptotes to graph a function.

Work each problem showing all supporting work ON THIS PAPER. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. No calculators.

1. On the graph below, mark slope and curvature using the proper derivative inequalities (as seen on page 7-1 and Example 1). Label the critical and inflection points.



2. The graph of the **derivative**  $f'(x)$  of a continuous function  $f(x)$  is shown. Find the following or state “none”.



(a) On what intervals is  $f$  increasing? Decreasing?

(b) Locate the  $x$ -value(s) where  $f$  has a local maximum? Minimum?

(c) On what intervals is  $f$  concave upward? Concave downward?

(d) Locate the  $x$ -value(s) where  $f$  has an inflection point.

3. For the function  $f(x) = x - 4\sqrt{x}$ , find the following if they exist *or state "none"*. (a) the zeroes (roots) (b) intervals of increase/decrease (c) location(s)  $(x, y)$  of local maximum(s)/minimum(s) (d) intervals of concavity and (e) location(s)  $(x, y)$  of any point(s) of inflection. You do not need to graph the function.