Chain Rule and Implicit Differentiation

(3.4, 3.5)

Prelab: Review Example 2 (p.200), Example 7 and formula 5 (p. 202), Example 2 (p. 210), Know Derivatives of Inverse Trigonometric Functions (p. 214)

I. Introduction: The differentiation rules you have learned allow you to find the derivative of many different functions including those created by combinations such as addition, multiplication or division. New functions can also be created by the composition of functions.

Given $f(x)$ and $g(x)$, we can create the composite functions, $f(g(x))$ and $g(f(x))$.

These new functions require the **Chain Rule** for differentiation:

(a) $\frac{d}{dx}[f(g(x))]$

(b) $\frac{d}{dx}[g(f(x))]$

When a function is the result of the composition of more than two functions, the chain rule for differentiation can still be used.

(c) $\frac{d}{dx}[f(g(h(x)))]$

Example 1:

(a) Given the composite function $y = e^{\sqrt{1-4x^2}}$, find y' and simplify.

(b) Find $f'(t)$, given $f(t) = [\tan(\cos(1-2t))]$.

II. Combining the power, multiplication, quotient and chain rules

Example 2:
(a) Given $g(x) = (6x^3 + 7)(1 - 5x^2)^3$, find $g'(x)$.

(b) Find $f'(x)$ given $f(x) = \left(\dfrac{x-4}{4x^3 + x}\right)^5$.

III. Implicit Differentiation

Introduction: The Chain Rule is used to find the derivative of a function defined implicitly rather than explicitly. When differentiating a function defined implicitly, treat the dependent variable as a function of the independent variable and apply the chain rule.

Notice, $\frac{d}{dx}\left[(2x+5)^3\right]=$

Now, suppose the "inside" function is expressed more generally. That is, let $f(x)=2x+5$. The inside function is still a function of x, so the chain rule applies.

$$\frac{d}{dx}\left([f(x)]^3\right)=$$

Our implicit functions are usually written with y as the dependent variable and since $y=f(x)$, we consider y^3 to be a composite function. We show the chain rule as follows:

$$\frac{d}{dx}\left(y^3\right)=$$

After each side of the equation is differentiated with respect to x, solve for $\frac{dy}{dx}$ (or y'). The function could, of course, contain different independent and dependent variables.

Example 3: Differentiate the implicit function $y^2 = x^2 + \sin(xy)$

Math 111 F24 Lab 4 Exercises Name: ___________________________ Section: ____ Score: _____

Work each problem ON THIS PAPER, showing all supporting work. You may use your textbook, lab and notes. Students may work cooperatively but each submits his/her own set of Lab Exercises. Do not use a calculator.

1. Compute each derivative. Consider a and b constants and y a function of x ($y = f(x)$). Do not simplify.

(a) $\frac{d}{dx}\left[(f(x))^a\right] =$

(b) $\frac{d}{dx}\left[b\sin^{-1}(f(x))\right] =$

(c) $\frac{d}{dx}\left[\frac{\ln(f(x))}{x}\right] =$

2. For each function below (i) find $f'(x)$ and simplify, (ii) determine where $f'(x) = 0$ if possible, if not state this and (iii) state the domain of $f'(x)$.

(a) $f(x) = \ln(x - \sqrt{x})$

(b) $f(x) = \frac{e^x}{1 - e^x}$

3. Find $\dfrac{dy}{dx}$ where, $x\sin y = xy^2$.

4. Find $\dfrac{ds}{dt}$, where $\sqrt{t+s} = 2s$. Simplify your final answer so it does not contain a complex fraction.