Calculating a Limit

(2.2, 2.3)

Prelab: Read sections 2.2 and 2.3 in your textbook. Review: Example 9 and Figure 17 (page 91), Example 3 (page 98), Example 6 (page 99), and Example 8 (page 121).

I. In calculating a limit, we attempt to answer the question, "what happens to $f(x)$ as x gets closer and closer to a?" In this lab, we investigate some limits and important related concepts.

Example 1. Consider $f(x)=\dfrac{x^4-1}{x^3-1}$. In order to evaluate $\lim\limits_{x\to 1} f(x)$ without a graph or a table of values, we use the limit laws and direct substitution property (2.3) as well as some algebra.

*Important concept 1: What does "direct substitution" lead to? _____

This result is an **indeterminate form of a limit.** This is another way of saying "you can't use direct substitution so you'll need to think of some clever algebra (or something else) in order to **determine** the limit." So, we do the following:

$$\lim_{x\to 1} f(x)=\lim_{x\to 1}\frac{x^4-1}{x^3-1}=$$

Two other indeterminate forms are $\dfrac{\pm\infty}{\pm\infty}$ and $\infty-\infty$. You will learn more indeterminate forms later.

*Important concept 2: The *functions* $f(x)=\dfrac{x^4-1}{x^3-1}$ and $f(x)=\dfrac{(x^2+1)(x+1)}{x^2+x+1}$ are not equal but the *limit of each function* (as $x\to 1$) yields the same value so the *limits* are equal.

II. One-Sided Limits

The limit found in Example 1, $\lim_{x\to 1}\frac{x^4-1}{x^3-1}=\frac{4}{3}$, indicates that as values of x approach 1 from both the left (values below 1) and right (values above 1), the value of $f(x)$ approaches $\frac{4}{3}$. This satisfies the theorem:

$$\lim_{x\to a} f(x)=L \text{ if and only if both } \lim_{x\to a^-} f(x)=L \text{ and } \lim_{x\to a^+} f(x)=L.$$

Example 2: Let $f(x)=\begin{cases} 2x, & x<-1 \\ x^2+1, & -1\le x<1 \\ (x-2)^3, & x\ge 1 \end{cases}$. Find the following:

(a) $\lim_{x\to 1^-} f(x)$ (b) $\lim_{x\to 1^+} f(x)$ (c) $\lim_{x\to 1} f(x)$

III. Infinite Limits

Remember the value "L" in the intuitive definition of a limit in your textbook? What if $f(x)$ does not approach a number, L, but instead gets infinitely large (or infinitely small)?

Example 3: Evaluate $\lim_{x\to 5^+}\frac{x+1}{x-5}$. (What does direct substitution lead to?) Justify any limit of $\pm\infty$ and state DNE where applicable.

Example 4: Evaluate $\lim_{x\to 0^+}\left(\frac{1}{x}-\ln x\right)$ (What does direct substitution lead to?) Justify any limit of $\pm\infty$ and state DNE where applicable.

FYI: The forms $\infty+\infty$ and $-\infty-\infty$, are determinate but the form, $\infty-\infty$, **is indeterminate.**

* Important Concept 3: A "limit" of infinity does not mean a limit exists. It is one way in which a limit fails to exist so include "DNE" (does not exist) along with either $\pm\infty$ when finding these "limits".

* Important Concept 4: Identifying an infinite limit gives important information about the behavior of a function.

* Important Concept 5: In both Example 3 and Example 4 above, the limit **does not exist**. This does not mean the limit is **indeterminate**. There is a difference!

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For each limit below: (a) State the indeterminate form present if any, otherwise, state "no indeterminate form". (b) Calculate each limit or justify why it does not exist (DNE, state this) showing intermediate steps and use of the limit laws. Final answers will either be a numerical value, ∞, or $-\infty$. Do not use L'Hospital's Rule (section 4.4) on this assignment.

You may use your textbook, lab, notes and peer collaboration (each student must submit their own assignment, however). **Do not use a calculator**, **table of values or graph unless otherwise instructed.** Include intermediate steps for full credit.

1. $\lim_{x \to 1^+} \frac{x^2 - 3x}{x - 1}$ Your intermediate step should include the limit of both the numerator and denominator.

2. $\lim_{x \to 1} \frac{\sqrt{3x+1} - 2}{x - 1}$

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3. $\lim_{h \to 2} \frac{3h^2 - 7h + 2}{2 - h}$

4. $\lim_{x \to 4} f(x)$, where $f(x) = \begin{cases} 2 - 3e^{4-x}, & x > 4 \\ 2 - \frac{3}{4}x, & x < 4 \end{cases}$ (You must show use of the proper theorem.)

5. $\lim_{x \to 0^+} \left(\ln x - \frac{1}{x} \right)$ Your intermediate step should include the limit of each term. You may wish to use the graph $y = \ln x$. Remember, no calculator.